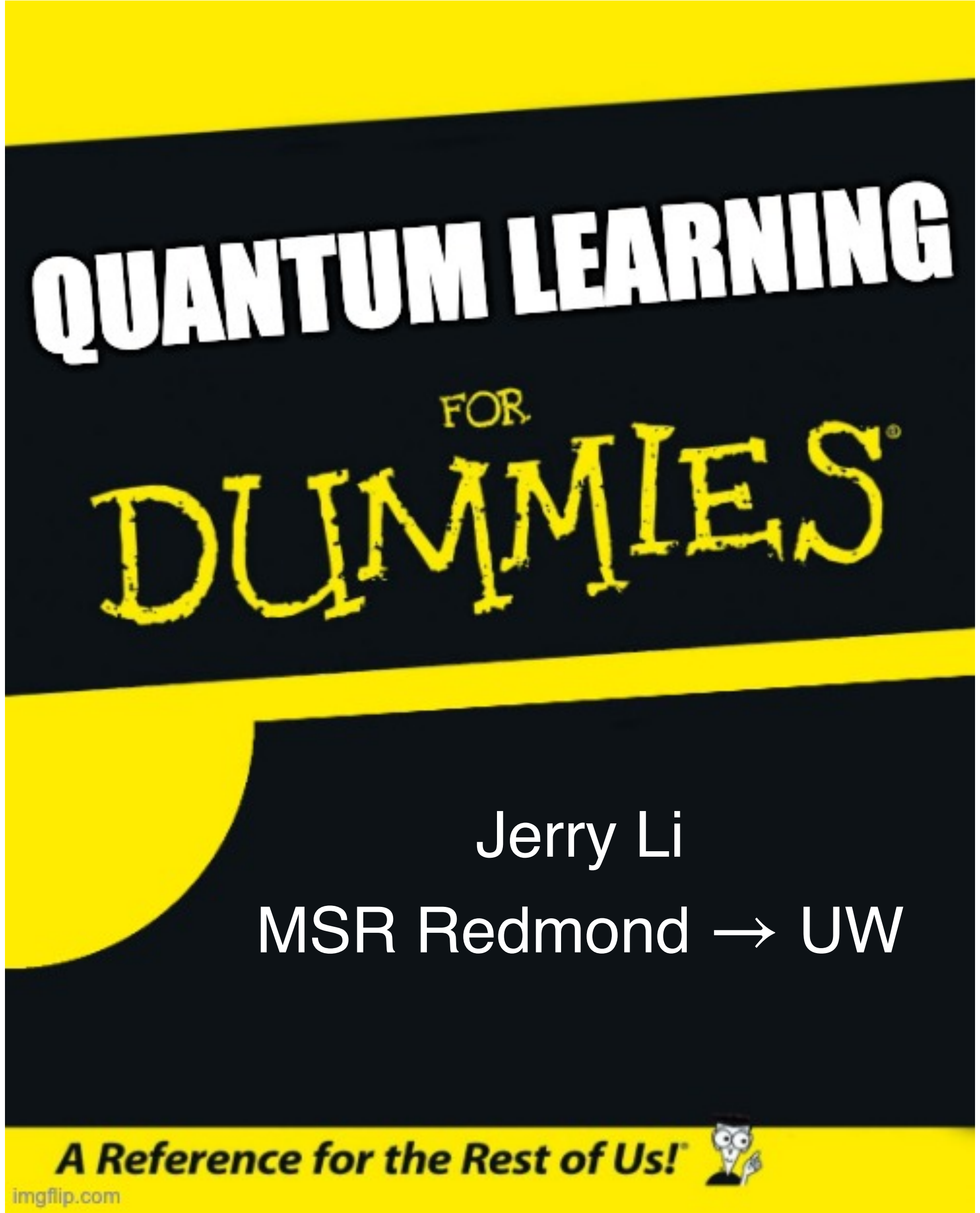
QUANTUM LEARNING
FOR
DUMMIES®
Jerry Li
MSR Redmond → UW
A Reference for the Rest of Us!
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Caveat

The goal of this talk is to describe the basic ideas of quantum learning to folks with TCS background, but maybe don't know quantum.

There will be $\leq \varepsilon$ physical intuition provided in this talk for why these are the "right" notions.

Instead, we will see how quantum learning arises naturally from a mathematical perspective.

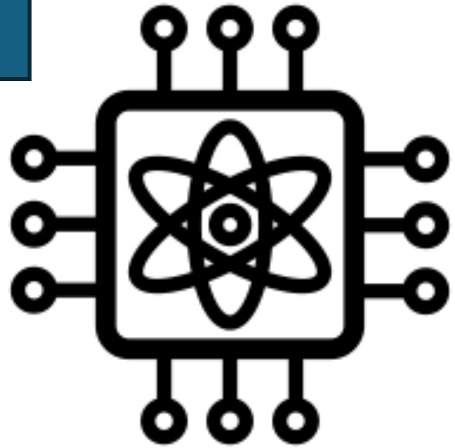
TL;DR just trust me

What is quantum learning?

How can we learn about quantum objects and phenomena?

- What is this object?
- Is it equal to some other object?
- What are its statistics?

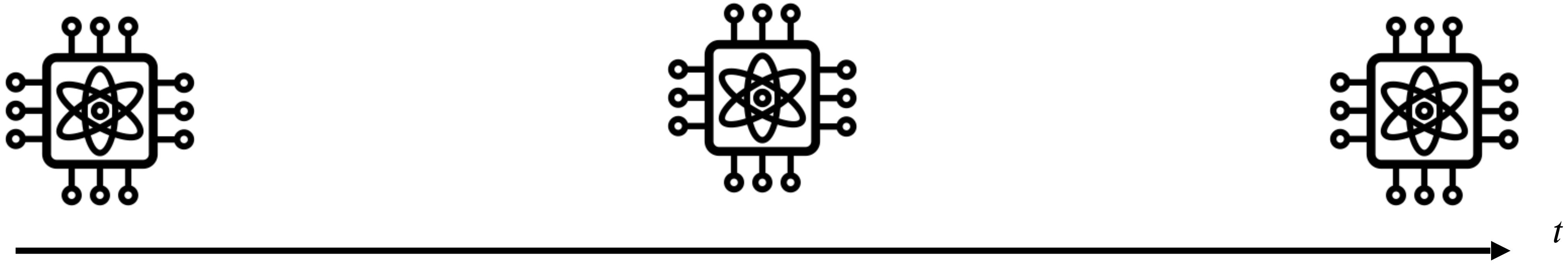
Arguably one of the mos estions!



ρ

(Online) quantum learning

In his lab, Prof. H sets up an experiment that can produce copies of his quantum state ρ one at a time.



Question: How many copies of ρ does he need to learn about it?

Why do we care?

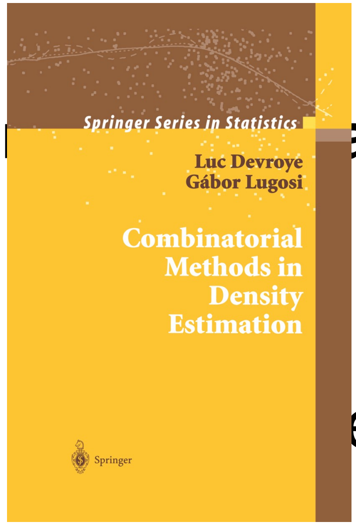
- Physical phenomena are **fundamentally quantum.**
- It is useful for **verifying** the outcome of quantum computation.
- This is the natural **non-commutative analogue** of distribution learning and testing.

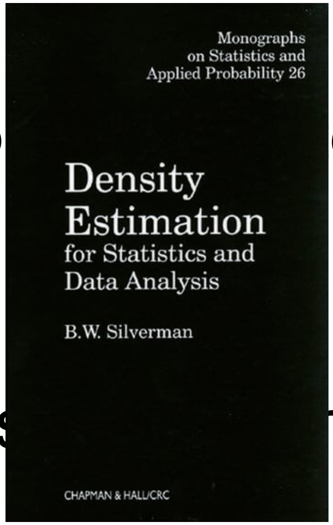
Classical testing and learning

Given n samples from an unknown distribution p, infer some property of p:

[Pea'00], [GGR'98], [GR'00], [BFRSW'00], [BFFKRW'01], [Pan'08], [VV'11], [Val'11], [VV'14], [CDVV'14], [CR'14], [CRS'15], [ADK'15], [JKHW'15], [DKN'15], [WY'15], [DK'16], [Gol'16], [WY'16], [Gol'17], [JKHW'17], [VV'17], [BC'18], [ADR'18], [DGPP'18], [JHW'18], [DR'19], [DGPP'19], [BCG'19], [DGKR'19], [AJM'20], [Can'20], [ACHSZ'20], [DGKPP'20], [WY'20], [CJKL'22], [CL'22], [CS'24]………

- **U**
- **I**
- **De** **atio** o error ε.





ents high probability)

Quantum testing and learning

Given n samples from an unknown distribution p, infer some property of p:

- **Uniformity testing:** Test if p is the uniform distribution or ε-far from it.
- **Identity testing:** Test if p is some given distribution q or ε-far from it.
- **Density estimation:** Learn p to error ε.

(All statements are with high probability)

Quantum testing and learning

Given n copies of an unknown quantum state ρ, infer some property of ρ:

- **Uniformity testing:** Test if p is the uniform distribution or ε-far from it.
- **Identity testing:** Test if p is some given distribution q or ε-far from it.
- **Density estimation:** Learn p to error ε.

(All statements are with high probability)

Quantum testing and learning

Given n copies of an unknown quantum state ρ, infer some property of ρ:

- **Mixedness testing:** Test if ρ is the maximally mixed state or ε-far from it.
- **Identity testing:** Test if p is some given distribution q or ε-far from it.
- **Density estimation:** Learn p to error ε.

(All statements are with high probability)

Quantum testing and learning

Given n copies of an unknown quantum state ρ, infer some property of ρ:

- **Mixedness testing:** Test if ρ is the maximally mixed state or ε-far from it.
- **State certification:** Test if ρ is some given state σ or ε-far from it.
- **Density estimation:** Learn p to error ε.

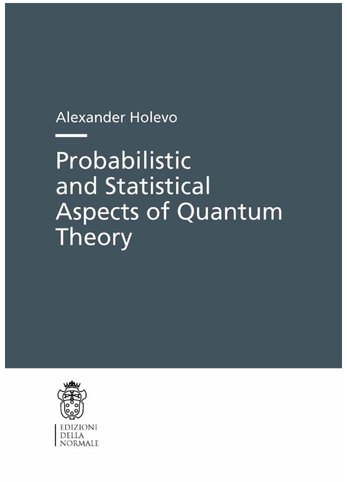
(All statements are with high probability)

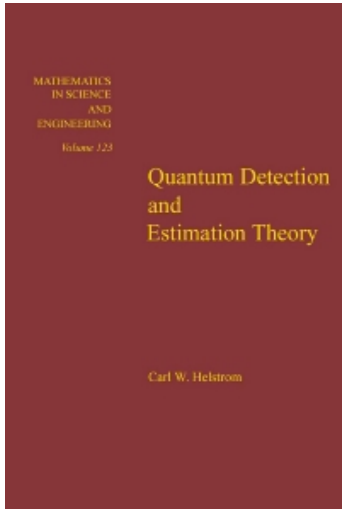
Quantum testing and learning

Given n copies of an unknown quantum state ρ, infer some property of ρ:

- **Mixedness testing:** Test if ρ is the maximally mixed state or ε-far from it.
  [O'Donnell, Wright'15]
- **State certification:** Test if ρ is some given state σ or ε-far from it.
  [Bădescu, O'Donnell, Wright'15]
- **State tomography:** Learn ρ to error ε.
  [Haah et al'16], [O'Donnell, Wright'16]

(All statements are with high probability)





Classical distributions

A distribution over d elements is specified by a vector $p \in \mathbb{R}^d$ on the simplex, i.e.

$$p_x \geq 0, \text{ for all } x = 1, \ldots, d$$
$$\sum p_x = 1$$

$p \quad =$

Quantum "distributions"

A mixed state over d elements is specified by a matrix $\rho \in \mathbb{C}^{d\times d}$ on the spectrahedron, i.e. a Hermitian matrix satisfying

$$p_x \geq 0, \text{ for all } x = 1, \ldots, d$$
$$\sum p_x = 1$$

$p \quad =$ 

Quantum “distributions”

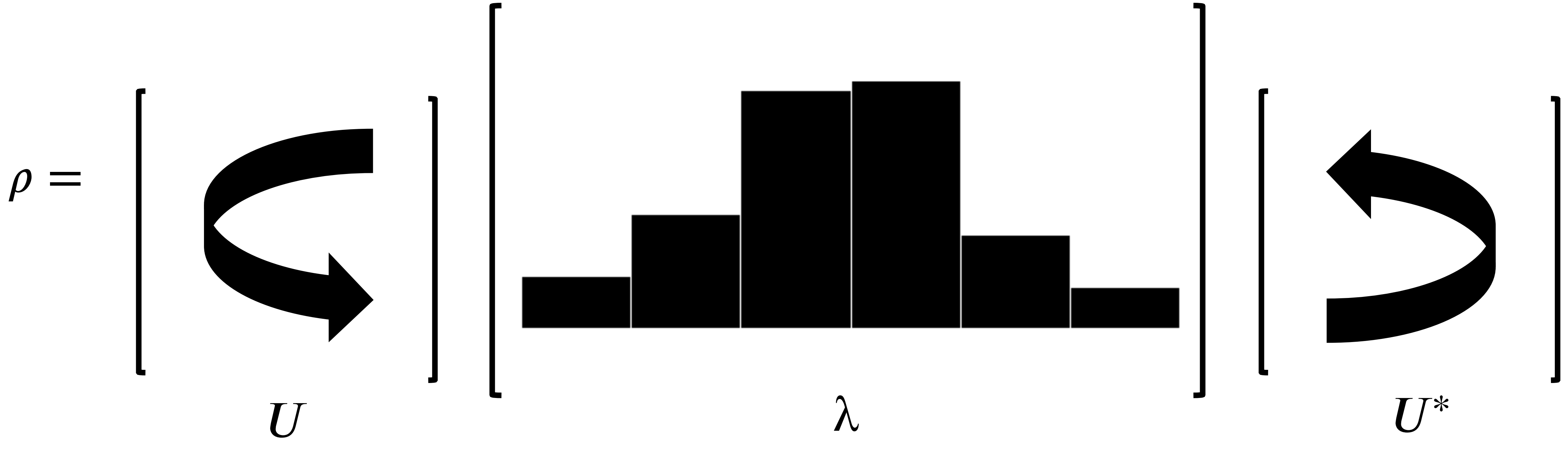
A mixed state over d elements is specified by a matrix $\rho \in \mathbb{C}^{d\times d}$ on the spectrahedron, i.e. a Hermitian matrix satisfying

$$\rho \succcurlyeq 0$$
$$\mathrm{tr}(\rho) = 1$$

$$p \quad =$$

Quantum “distributions”

By the spectral theorem, $\rho = U\lambda U^*$, where U is a unitary (rotation) matrix, and λ is a diagonal matrix with nonnegative entries which sum to 1.



Quantum distributions and pure states

More formally,

$$\rho = \sum_{i=1}^{d} \lambda_i \phi_i \phi_i^\dagger = \sum_{i=1}^{d} \lambda_i |\phi_i\rangle\langle\phi_i|$$

where the $|\phi_i\rangle$ are *pure states*, i.e. d-dimensional unit vectors, and λ_i are nonnegative and satisfy $\sum \lambda_i = 1$.

In this way, ρ can be thought of as a probability distributions over pure states.

Braket notation: be not afraid!

$|x\rangle$ is just a column vector with label x, $\langle x|$ is the corresponding row vector, i.e. $x^\dagger$.

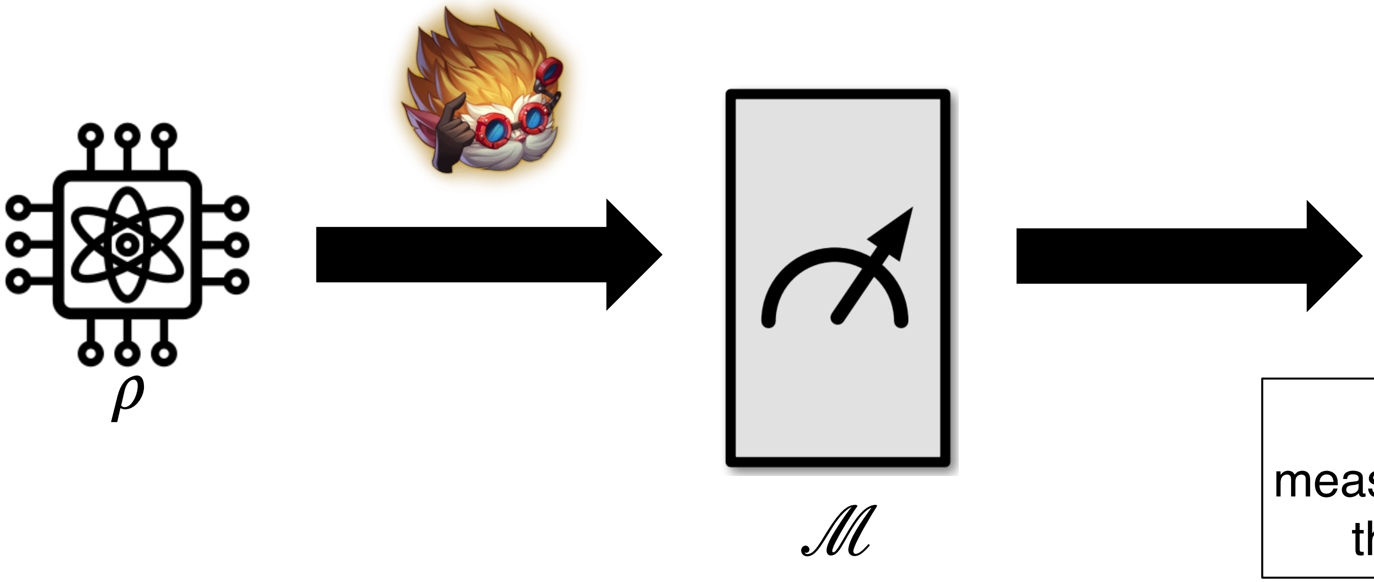
$$|x\rangle\langle x| = xx^\top$$
$$\langle x|y\rangle = x^\dagger y$$
$$\langle x|\rho|x\rangle = x^\dagger \rho x$$
$$|x_1\rangle|x_2\rangle = |x_1x_2\rangle = x_1 \otimes x_2$$

Folks kinda stick in whatever they want for x—it's just an index!

Quantum measurement

One does not simply sample from a quantum state!

To interact with a quantum state, one must specify a measurement $\mathscr{M}$.



$$X \sim p(\rho, \mathscr{M})$$

p is a classical distribution over measurement outcomes that depends on the relationship between ρ and $\mathscr{M}$

Measuring quantum distributions

A measurement is a collection of $d \times d$ matrices $\mathscr{M} = \{M_1, \ldots, M_\ell\}$ such that:

1. $M_i \succcurlyeq 0$ for all $i = 1, \ldots, \ell$, and
2. $M_1 + \ldots + M_\ell = I$

$\mathscr{M}$ is referred to as a positive-operator valued measure (POVM).

The outcome of measuring ρ using this POVM is that we observe M_i with probability

$$p_i = \mathrm{tr}(\rho M_i)$$

Measurement and collapse

Additionally, measuring a state *collapses* the state.

Formally, if $\mathscr{M} = \{M_1, \ldots, M_\ell\}$, and $M_i = B_i B_i^\dagger$ where the B_i are known as the *Kraus operators,* then if you measure ρ and get outcome M_i, the state collapses to a state

$$\rho \to \frac{B_i^\dagger \rho B_i}{\mathrm{tr}(B_i^\dagger \rho B_i)}$$

Intuitively, the more informative the measurement is, the more destructive it is to the state.

Example 1

Consider the trivial POVM $\mathscr{M} = \{I\}$.

If we measure a state ρ with this POVM, we get the outcome I deterministically, i.e. we learn nothing.

ρ also trivially collapses, i.e. $\rho \to \rho$.

Example 2

Given a classical distribution p over $\{1, \ldots, d\}$, form

$$\rho = \begin{bmatrix} p_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & p_d \end{bmatrix}$$

and consider the POVM $\mathscr{M} = \{ |e_1\rangle\langle e_1|, |e_2\rangle\langle e_2|, \ldots, |e_d\rangle\langle e_d| \}$.

Then, the probability of seeing the i^{th} outcome is p_i, so this recovers the classical setting.

Also, if we see outcome i, then $\rho \to |e_i\rangle\langle e_i|$, i.e. ρ collapses completely.

Exponentiality in quantum learning

A single qubit quantum state is described by a $|\phi\rangle \in \mathbb{C}^2$

So, n non-interacting qubits is described by $|\phi_1\rangle \ldots |\phi_n\rangle \in \mathbb{C}^{2^n}$, and a more general n qubit state is an arbitrary element of this space.

In general, we are working with "distributions" over exponentially large spaces, i.e. $d = 2^n$, so ideally we want runtimes like $\mathrm{poly}\log(d)$

How can we efficiently learn interesting things in this very high dimensional space?

Exponentiality in quantum learning (cont.)

One cannot avoid $\mathrm{poly}(d)$ sample complexity and runtime for learning general quantum states.

But we don't care about learning everything about all quantum states!

- Learning restricted properties? (shadow estimation)
- Learning restricted classes of states?

See Anurag's talk!

Exponentiality in quantum learning (cont.)

Because the systems are over exponentially large domains, we also can’t afford to perform arbitrary measurements.

We need to design quantum circuits that efficiently prepare the measurements that we wish to make.

What are some ways of efficiently preparing interesting classes of quantum circuits/measurements? → see Jonas’s talk!

Quantum systems and Hamiltonians

The time evolution of a quantum system is defined by an operator known as a *Hamiltonian*.

Finding ground states of Hamiltonians $\Longleftrightarrow$ solving CSPs

Gibbs sampling $\Longleftrightarrow$ sampling graphical models

Learning local Hamiltonians $\rightarrow$ see Ewin's talk!

Quantum learning in the real world

So we don't have general, fault-tolerant quantum computation yet.

But we do have intermediate-scale devices.

- Google Sycamore (2019) has ~50 qubits
- IBM Condor (2023) has ~1100 qubits

These are so-called noisy intermediate scale quantum (NISQ) devices.

Can we design learning algorithms for such devices? → See Xun's talk!

Resource constrained quantum learning

One key resource that NISQ devices are very constrained on is **quantum memory:** the ability to persistently store quantum information.

My (and my co-organizers) work:

Can we develop a theory for learning on quantum memory-bounded devices?

Resource constrained quantum learning

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RESEARCH ARTICLE | QUANTUM COMPUTING

Quantum advantage in learning from experiments

HSIN-YUAN HUANG, MICHAEL BROUGHTON, JORDAN COTLER, SITAN CHEN, JERRY LI, MASOUD MOHSENI, HARTMUT NEVEN, RYAN BABBUSH, RICHARD KUENG, JOHN PRESKILL, AND JARROD R. MCCLEAN fewer Authors Info & Affiliations

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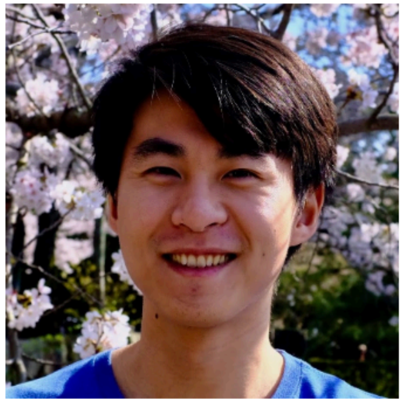
One of the first **unconditional** demonstrations of quantum advantage **ever!**

Shameless plug

UW has a fledgling quantum computation group!

Andrea Coladangelo



Jerry Li

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Come work with us!