Fast Algorithms for Segmented Regression

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General Motivating Question

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"As data grows, it may be beneficial to consider faster inferential algorithms, because the increasing statistical strength of the data can compensate for the poor algorithmic quality." [Jor13]

Outline

- Introduction
- The exact algorithm
- Our algorithm
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Linear regression

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Goal: Find a linear $\ell(\boldsymbol{x})$ minimizing

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We consider *fixed design* regression: we assume the $\boldsymbol{x}^{(i)}$ are fixed, and the only randomness is over the $\epsilon^{(i)}$.

The least squares estimator

Definition (Least squares estimator)

The least squares estimator, denoted $\widehat{\ell}^{\mathrm{LS}}$, is given by:

$$\widehat{\ell}^{\mathrm{LS}} \overset{def}{=} \underset{\substack{\ell \text{ linear}}}{\arg\min} \frac{1}{n} \sum_{i=1}^{n} (y^{(i)} - \ell(\boldsymbol{x}^{(i)}))^2 \ .$$

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How well does it recover the ground truth ℓ^*?

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Theorem

Let $\widehat{\ell}^{\mathrm{LS}}$ be as above. Suppose that $\epsilon^{(i)} \sim \mathcal{N}(0, \sigma^2)$. Then with high probability,

$$\mathrm{MSE}(\widehat{\ell}^{\mathrm{LS}}) = O\left(\sigma^2 \frac{d}{n}\right) .$$

Moreover, $\widehat{\ell}^{\mathrm{LS}}$ can be computed in time $O(nd^2)$.

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More recent work (see e.g. [CW13]) gets even faster theoretical runtimes.

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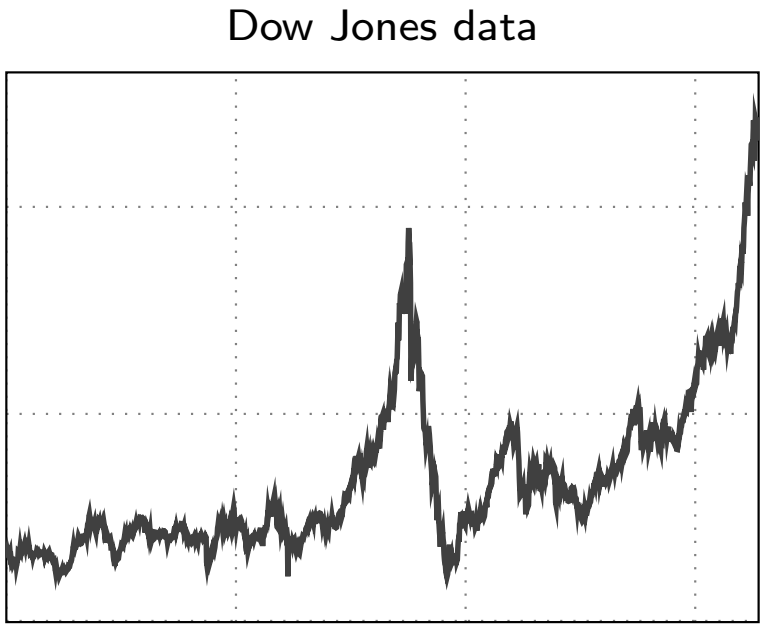

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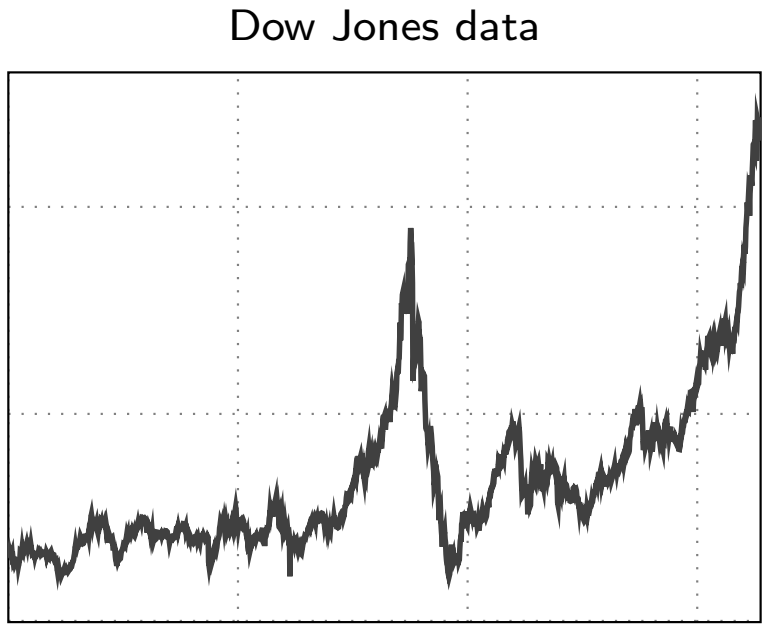

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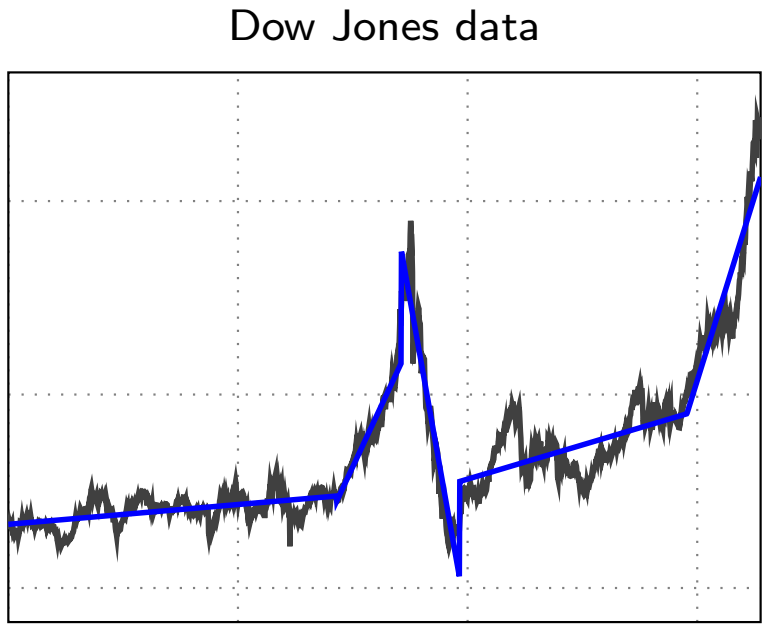

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A: Model it with a *piecewise linear* fit!

Segmented Regression

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A function $f : \mathbb{R}^d \to \mathbb{R}$ is k-piecewise linear if there exists a partition of $\mathbb{R}$ into k intervals $I_1, \ldots, I_k$ so that for all j, the function f is linear restricted to the set of $\boldsymbol{x} \in \mathbb{R}^d$ so that $\boldsymbol{x}_1 \in I_j$.

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Segmented Regression [BP98, YP13]

Given a data set $(\boldsymbol{x}^{(1)}, y^{(1)}), \ldots, (\boldsymbol{x}^{(n)}, y^{(n)})$ so that

$$y^{(i)} = f(\boldsymbol{x}^{(i)}) + \epsilon^{(i)} \ ,$$

where $\epsilon^{(i)}$ are independent noise and f is k-piecewise linear, recover f in MSE.

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$$\mathrm{MSE}(\widehat{f}_k^{\mathrm{LS}}) = O\left(\sigma^2 \frac{kd}{n}\right) \ .$$

Moreover, this rate is optimal.

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The algorithm took $\Theta(1)$ minutes to run for 10^4 samples, and $\Theta(1)$ hours to run for 10^5 samples.

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Our Results

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Formally...

Theorem

There is an $4k$-piecewise linear estimator $\widehat{f}$ which can be computed in time $O(n(d^2 + k))$ so that w.h.p.

$$\mathrm{MSE}(\widehat{f}) \leq \widetilde{O}\left(\sigma^2 \frac{kd}{n} + \sigma\sqrt{\frac{k}{n}}\right) .$$

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Speedup: $\widetilde{O}(\min\left(\frac{kd}{\epsilon}, kd^2\right))$

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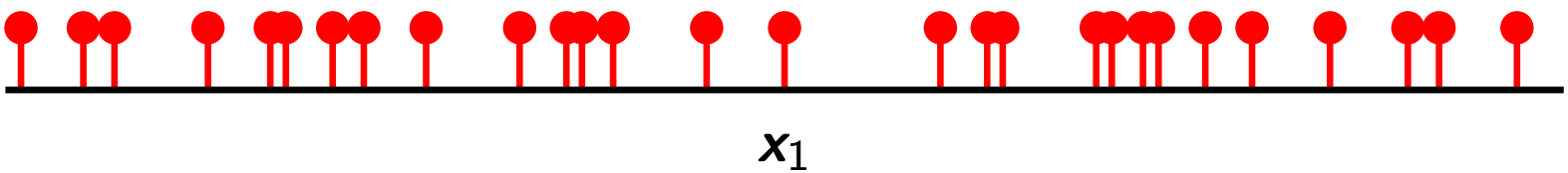
- **Output** The linear least squares fit over each interval in $\mathcal{I}_j$

Example: $k = 2$



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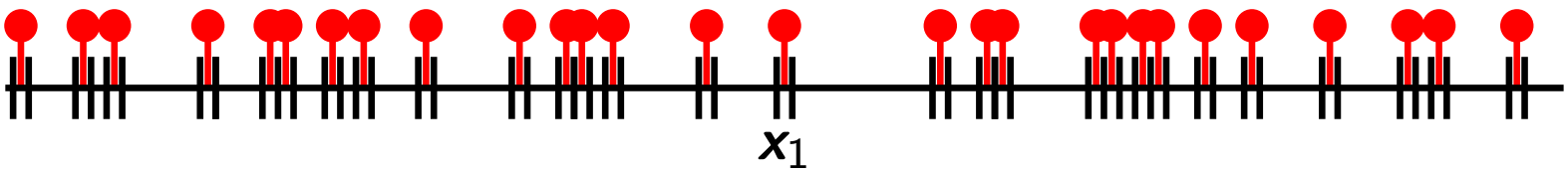

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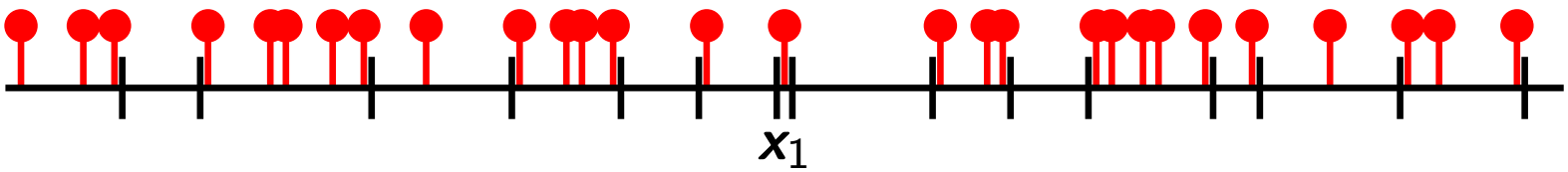

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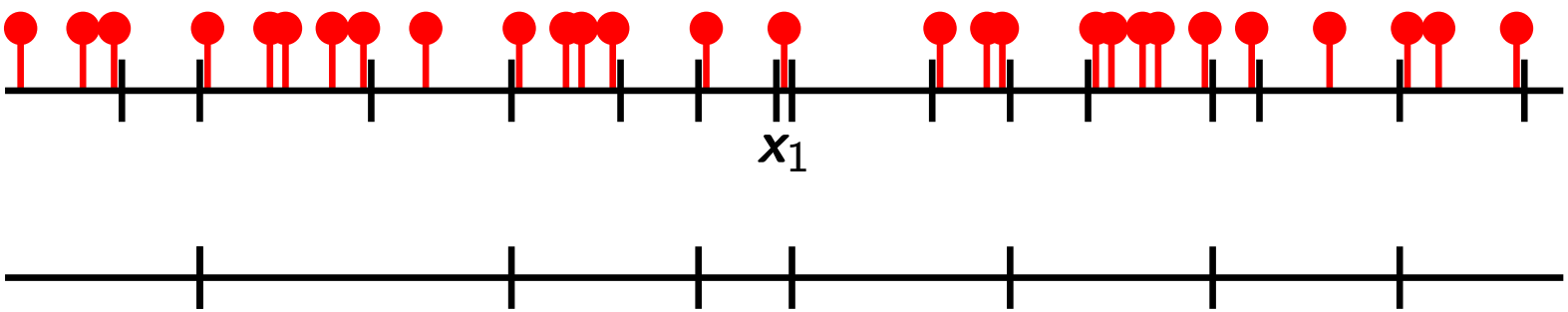

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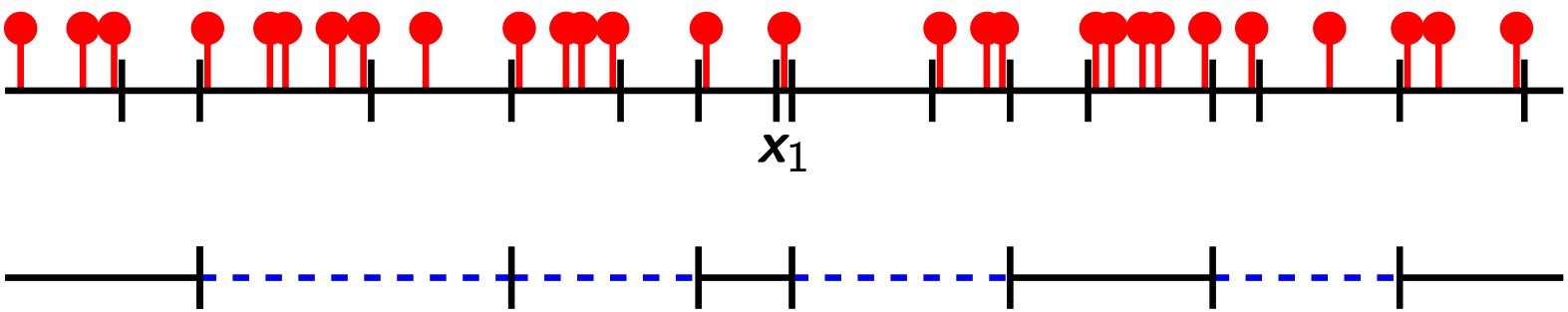

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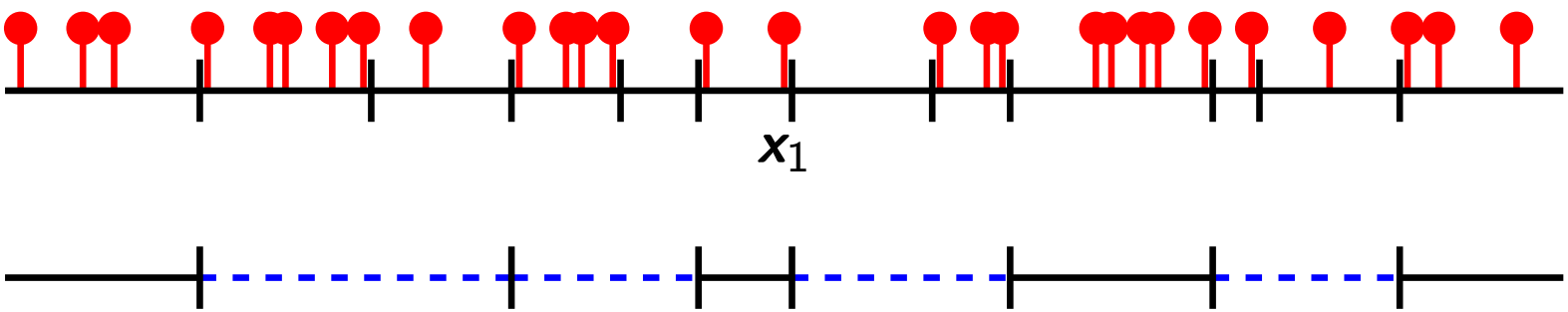

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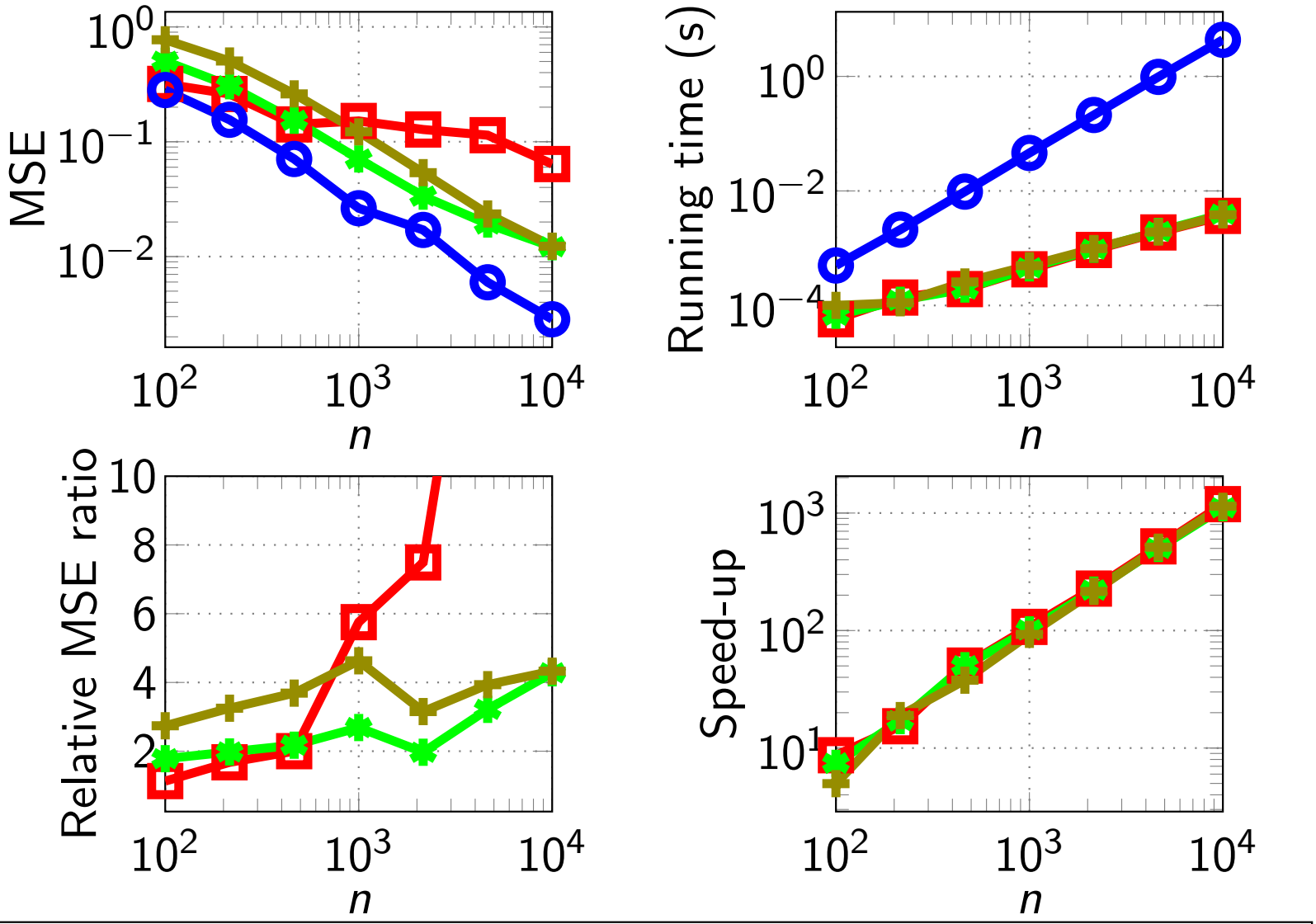
- Can show that our algorithms are robust to model misspecification.

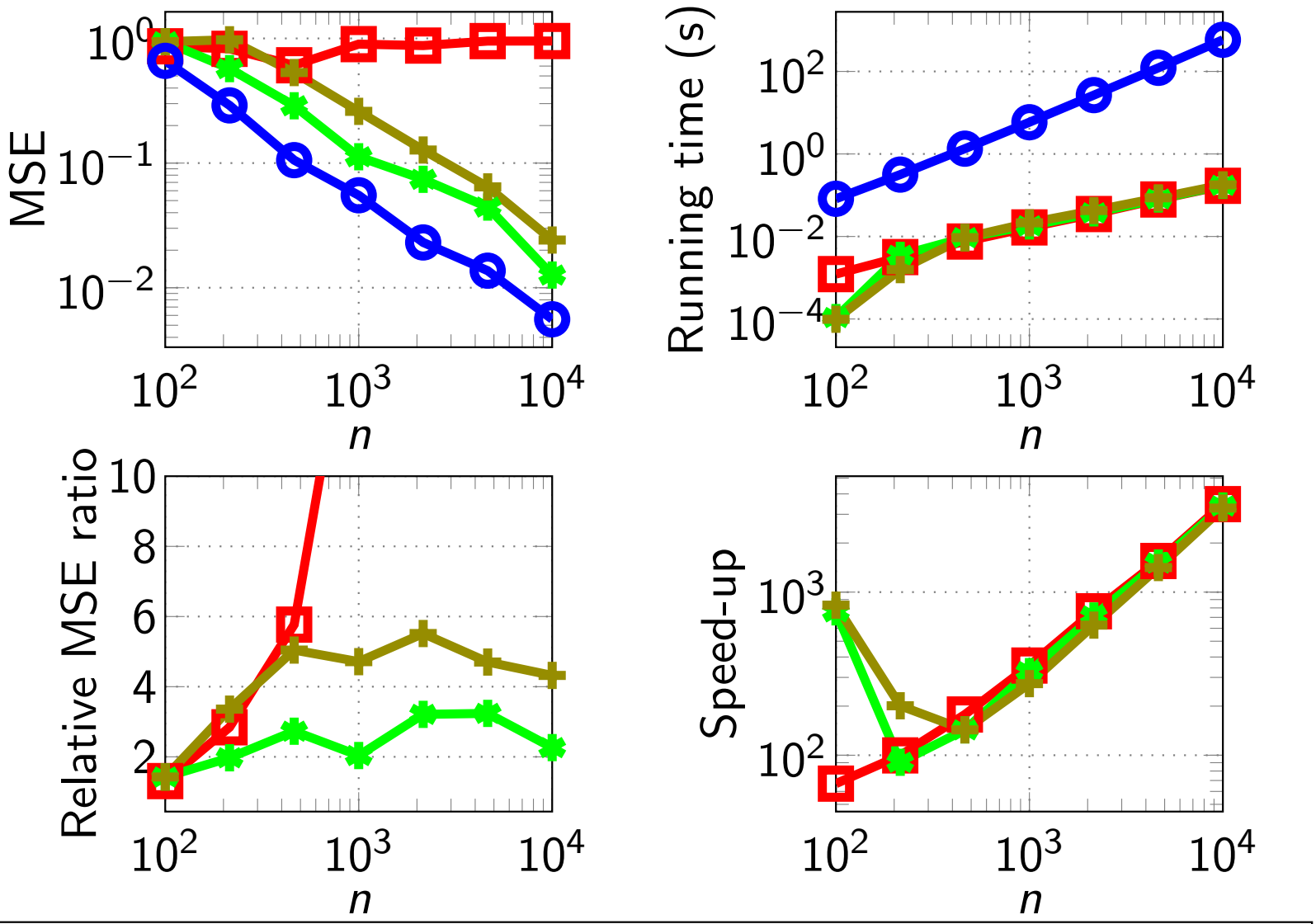
Outline

- Introduction
- The exact algorithm
- Our algorithm
- **Experiments**

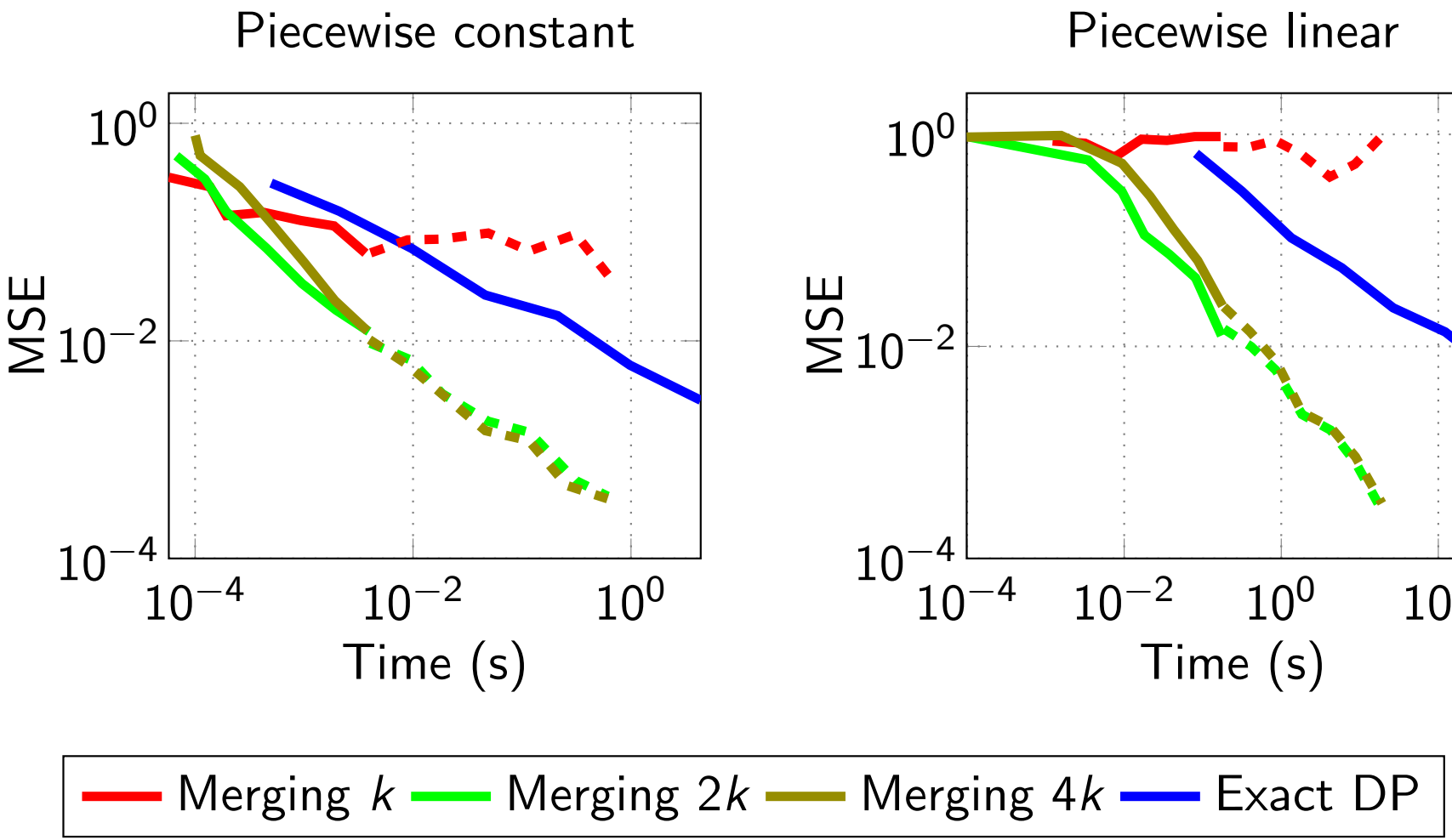
Experiments: piecewise constant



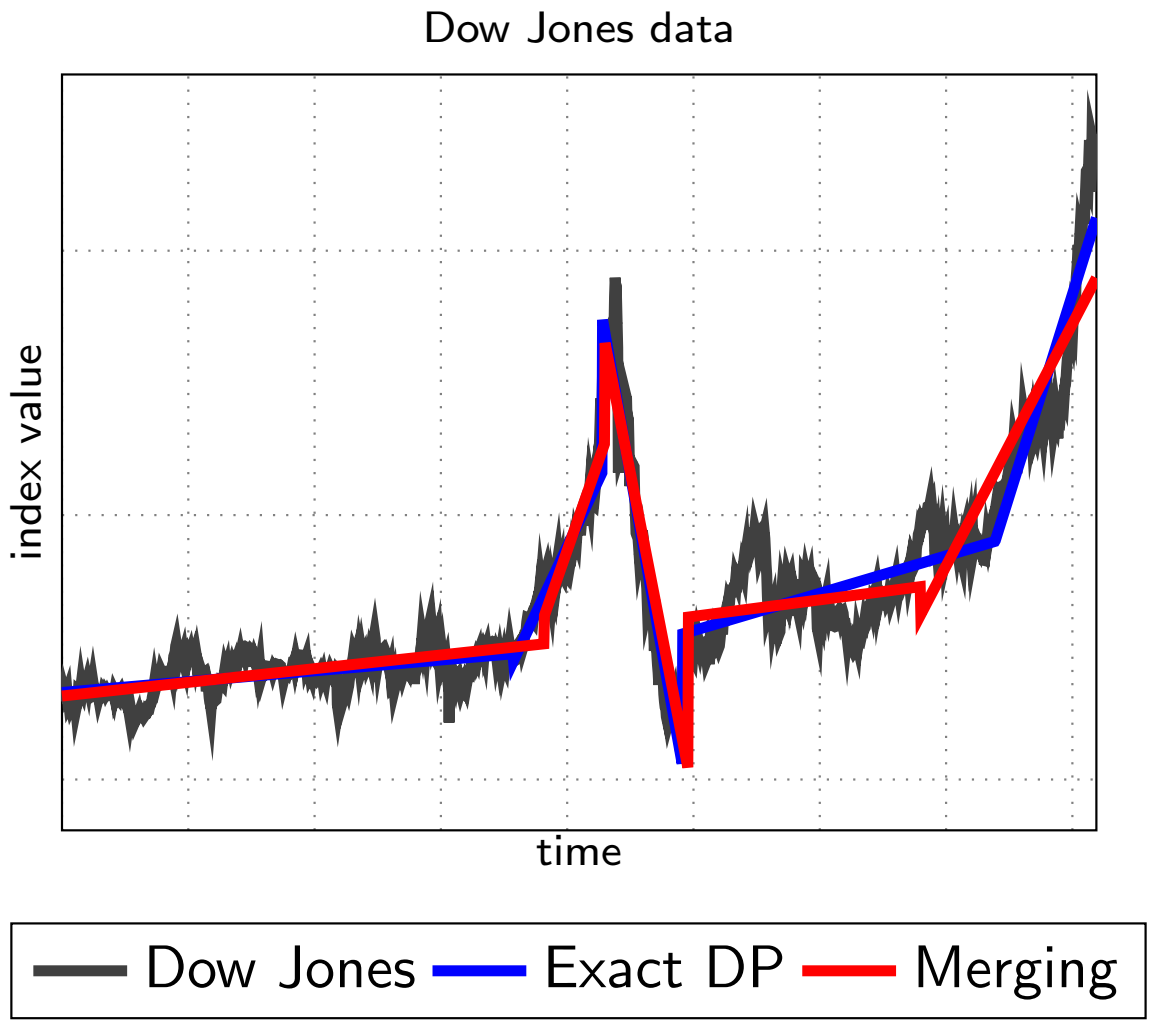
Experiments: piecewise linear



Experiments: time vs error trade-off



Experiments: real data



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- Given enough data, we get the same MSE $100\times$ faster

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Thank you!