Fast and Near-Optimal Algorithms for Approximating Distributions by Histograms

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Motivating Example

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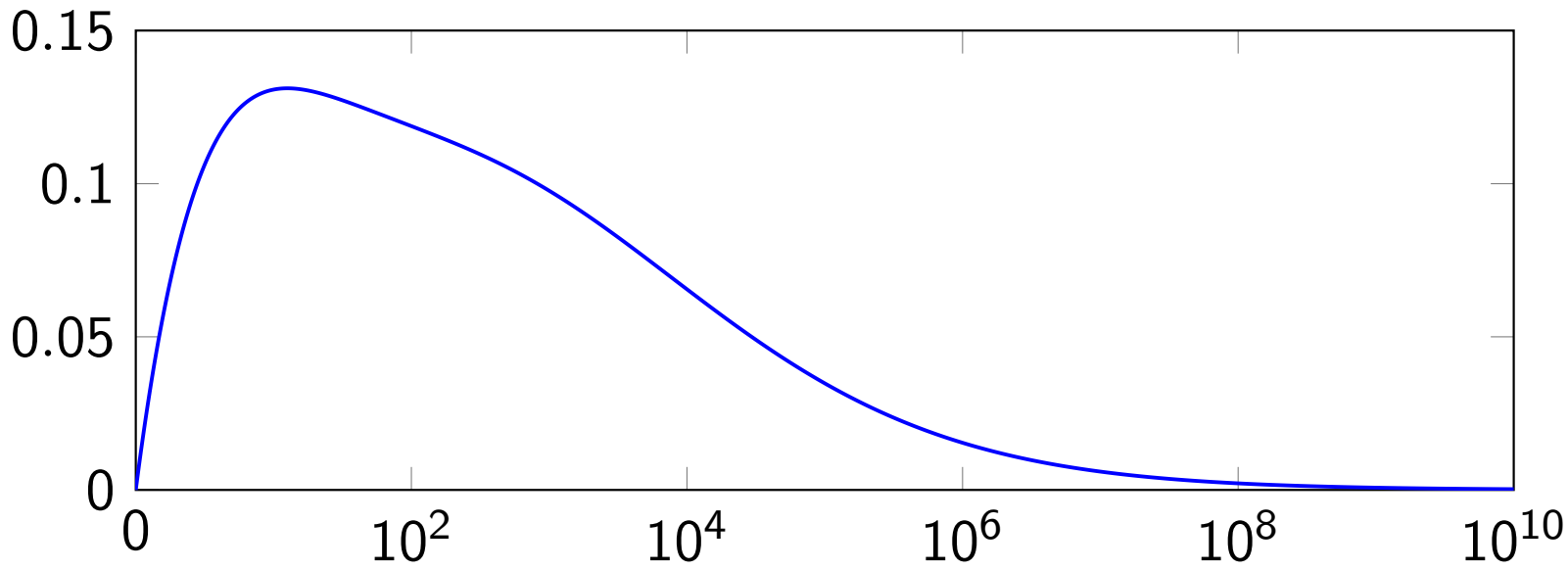

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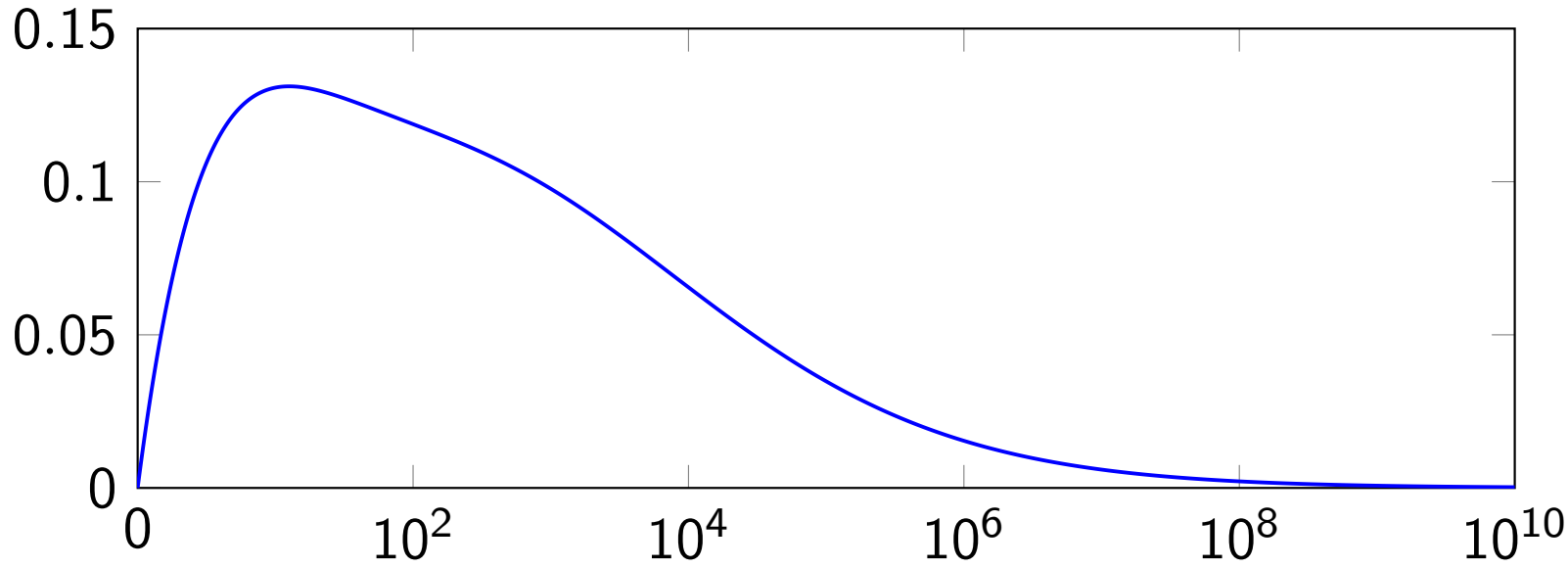

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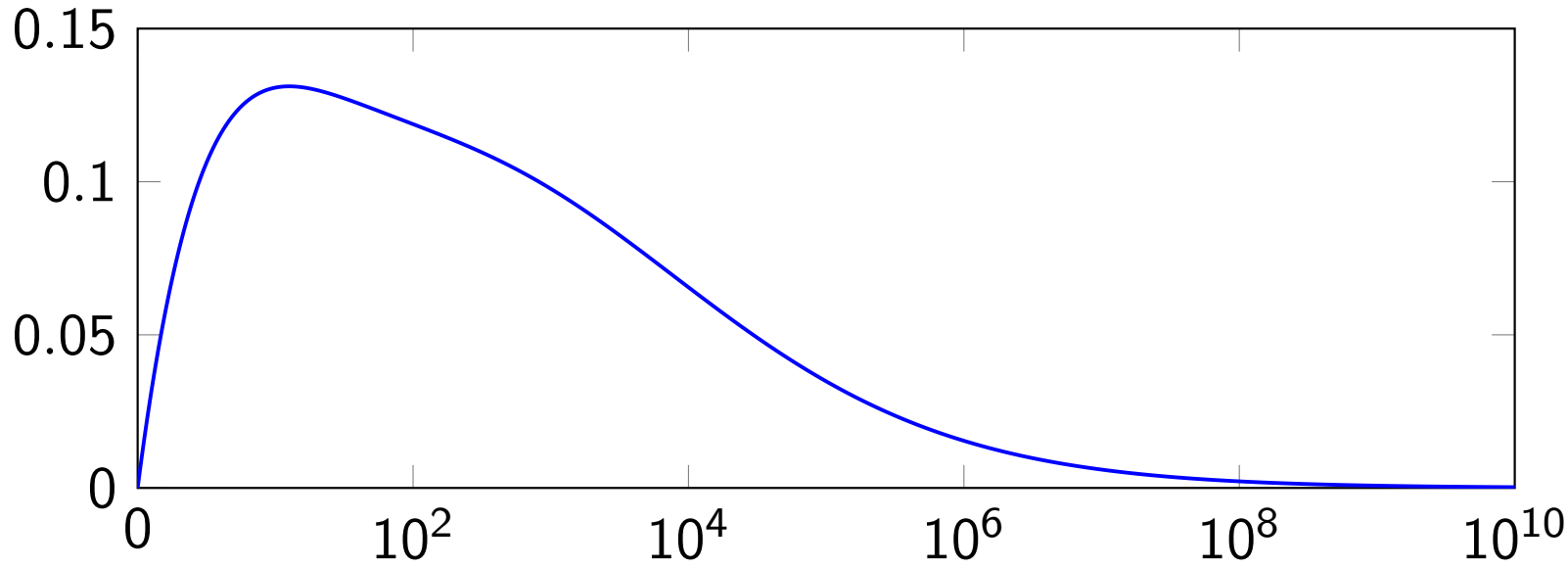

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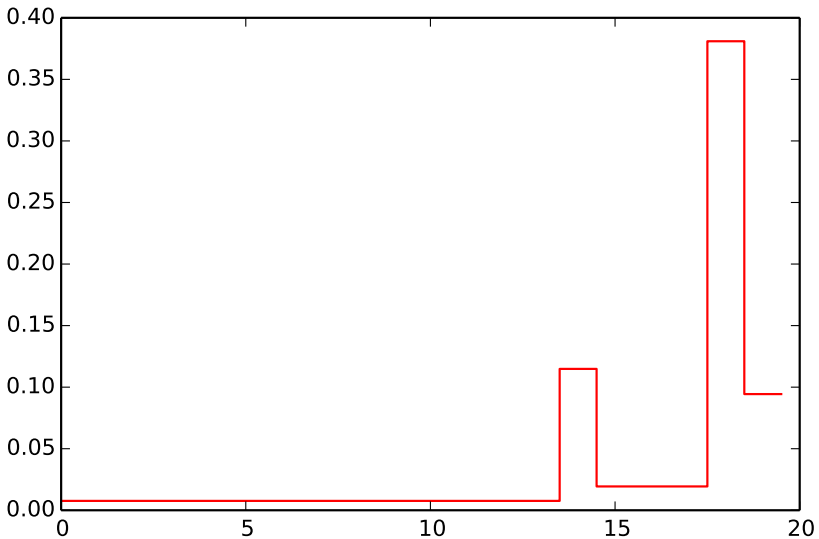

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Instead of storing f, efficiently find and store k-histogram h (for some small value of k) so that

$$\|f - h\|_2^2 \text{ is small,}$$

where $\|g\|_2^2 = \sum_{i=1}^{n} g(i)^2$ is the Sum-Squared Error or *V-optimal* error [IP95].

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i.e. Draw *independent samples* from f, and use them to recover a good k-histogram approximation to f

Formal Problem Statement

Given $k \in \mathbb{N}, \epsilon > 0$, and independent samples from an unknown distribution f supported on $[n] = \{1, \ldots, n\}$, recover a k-histogram h so that w.h.p.,

$$\|f - h\|_2^2 \leq \mathrm{OPT}_k(f) + \epsilon \,,$$

where for any function $q : [n] \to \mathbb{R}$,

$$\mathrm{OPT}_k(q) = \inf_{\text{h: } k\text{-histogram}} \|q - h\|_2^2 \,.$$

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Given $k \in \mathbb{N}, \epsilon > 0$, and independent samples from an unknown distribution f supported on $[n] = \{1, \ldots, n\}$, recover a $\alpha \cdot k$-histogram h so that w.h.p.,

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Gold Standard: Algorithm which achieves $\alpha = \beta = 1$ which takes an *information-theoretically* optimal number of samples, and runs in time *linear* in the number of samples taken.

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When given complete access to f:

Algorithm	Runtime	α	β
Basic DP [JKM+98]	$O(kn^2)$	1	1
Greedy Dual [JKM+98]	$O\left(n \log \mathrm{OPT}_k(f)\right)$	3	3
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Our algorithm is sample optimal (up to constants) and runs in linear time.

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Key lemma:

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If $m = O(\frac{1}{\epsilon})$, then $\|f - \hat{f}_m\|_2^2 \leq \epsilon$ with probability $99/100$.

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This reduces to a completely deterministic problem!

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An algorithm which, given $k \in \mathbb{N}$ and $q : [n] \to \mathbb{R}$ supported on m elements, runs in time $O(m)$, and outputs a $5k$-histogram h so that

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- Draw $m = O(1/\epsilon)$ samples and form the empirical $\hat{f}_m$.
- Run the above algorithm on $\hat{f}_m$. □

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Let $q : [n] \to \mathbb{R}$, and and let $I \subseteq [n]$ be an interval.

- Let $\overline{q}_I$ be the constant function on I which is identically $\frac{1}{|I|} \sum_{i \in I} q(I)$. We call this the *flattening* of q over I.
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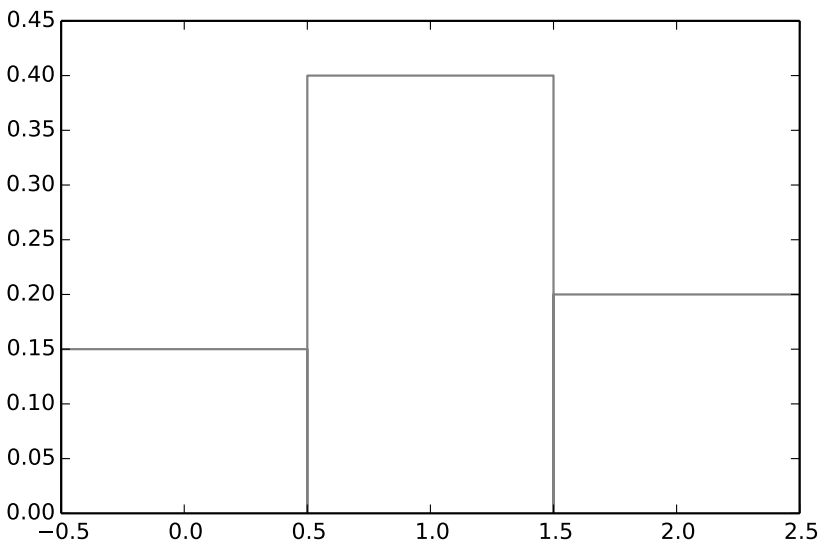

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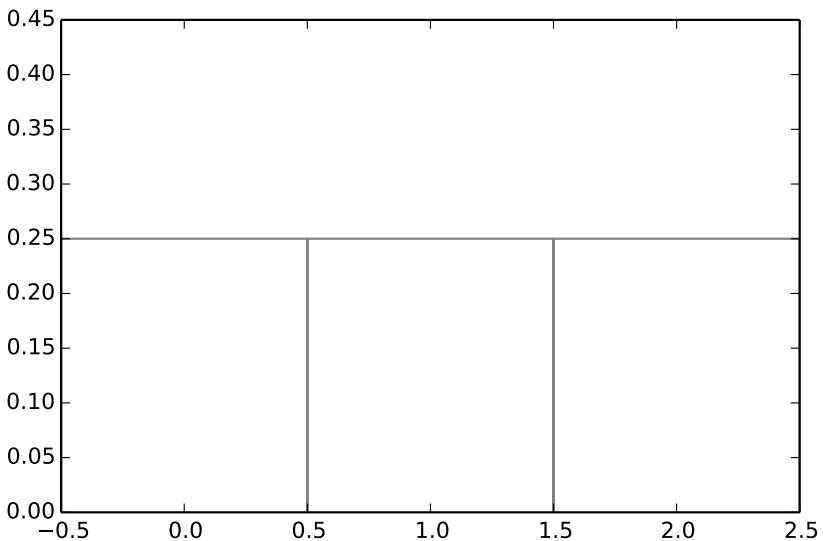

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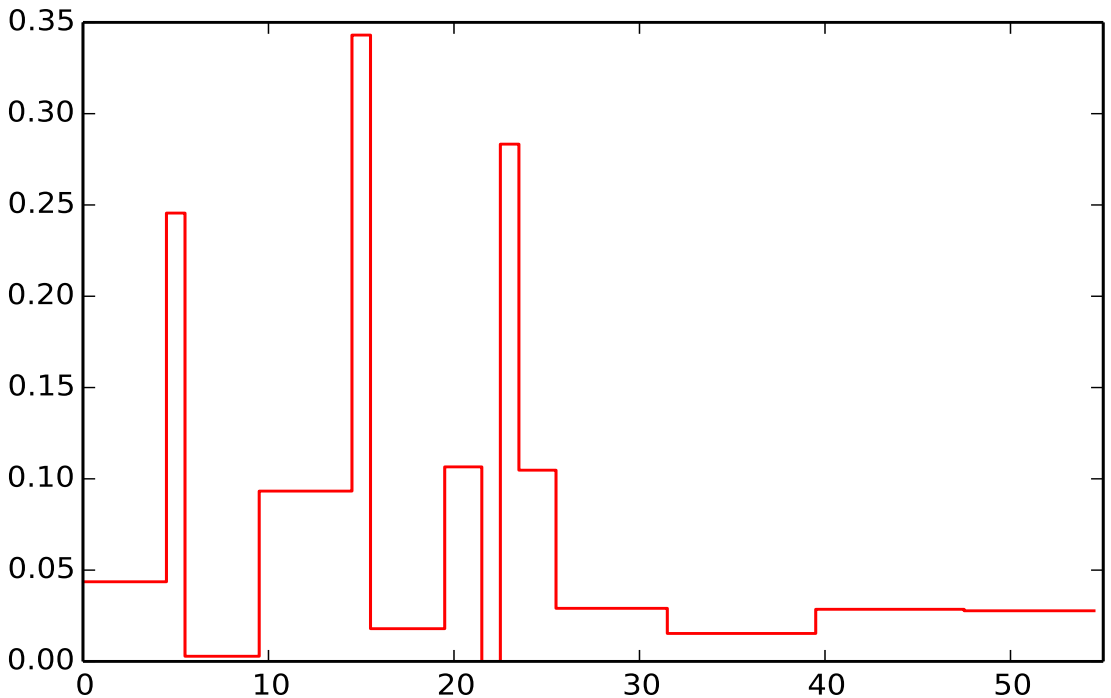

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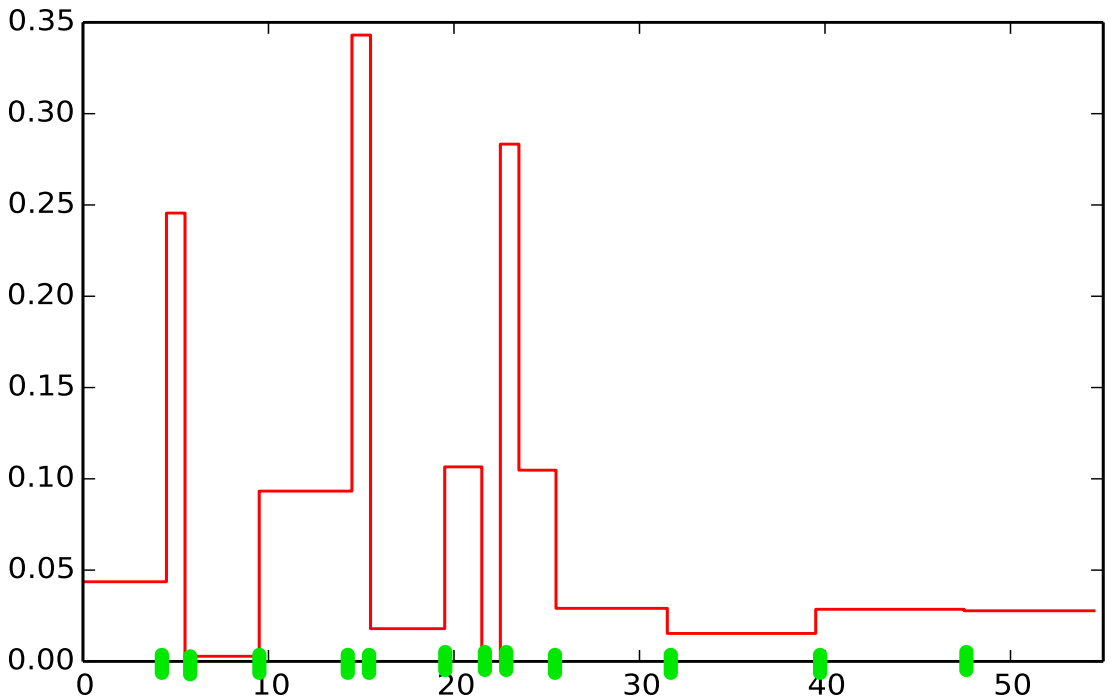

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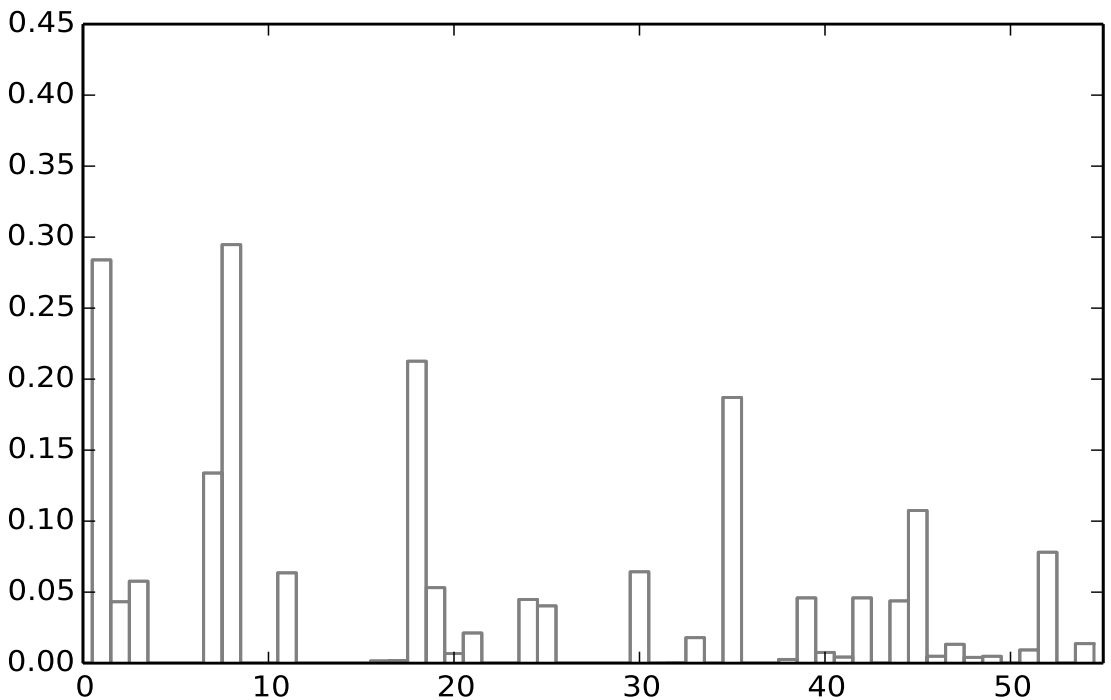

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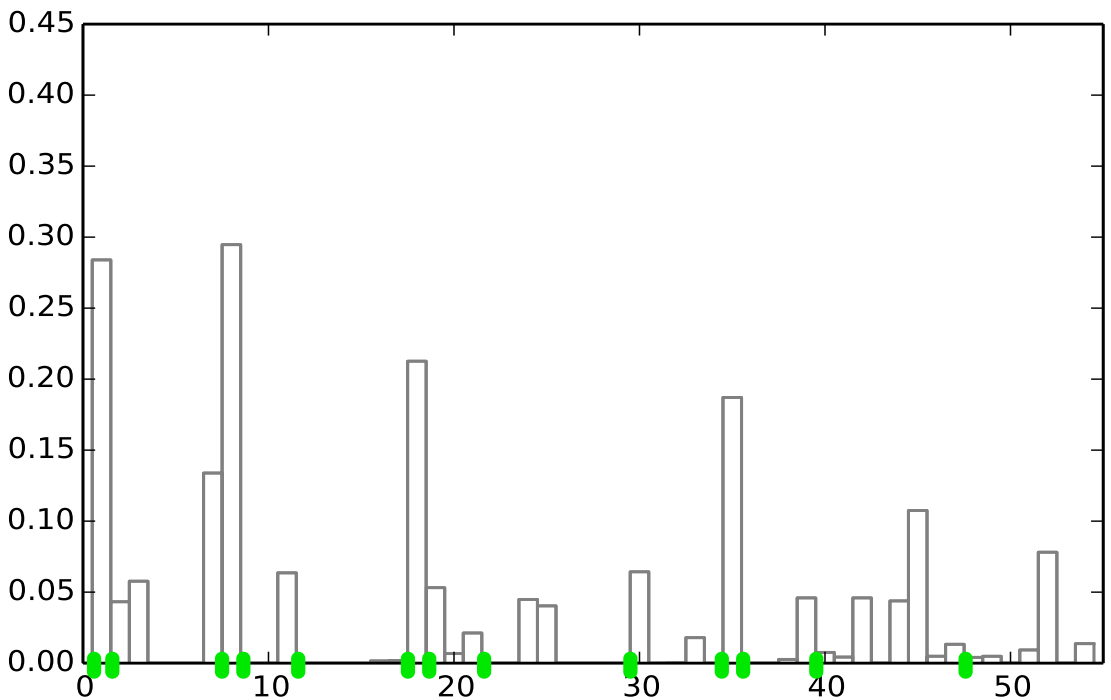

Partitions

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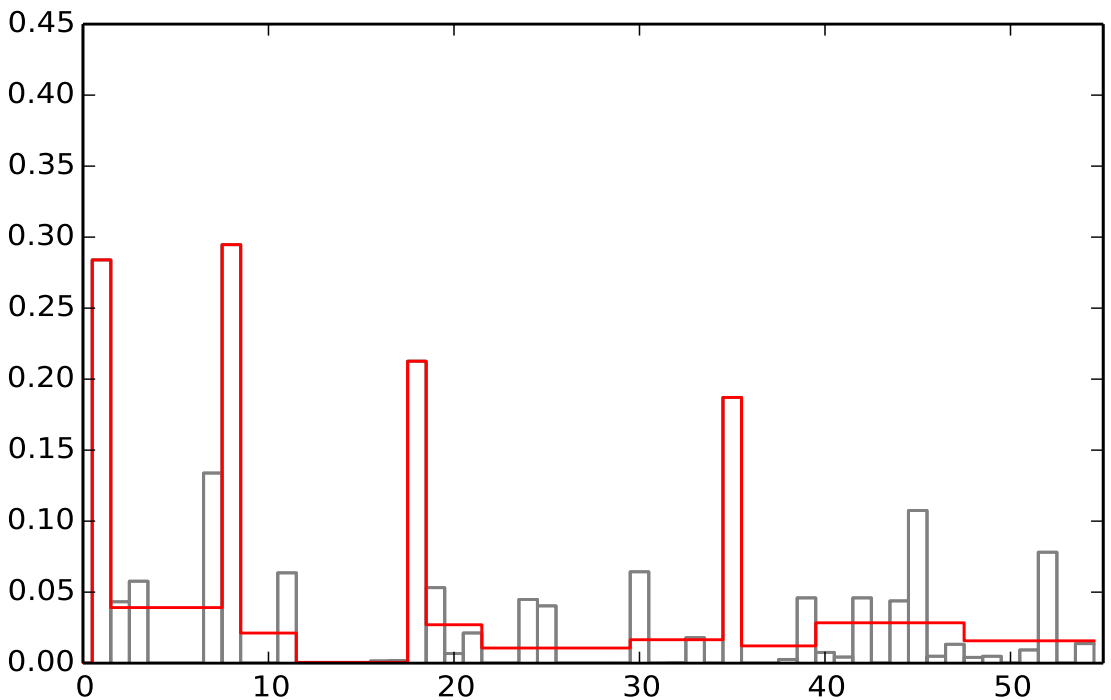
A *partition* of $[n]$ is a set of disjoint intervals $I_1, \ldots, I_r$ so that $\bigcup I_i = [n]$.

- Any partition induces a unique k-histogram (for our purposes)



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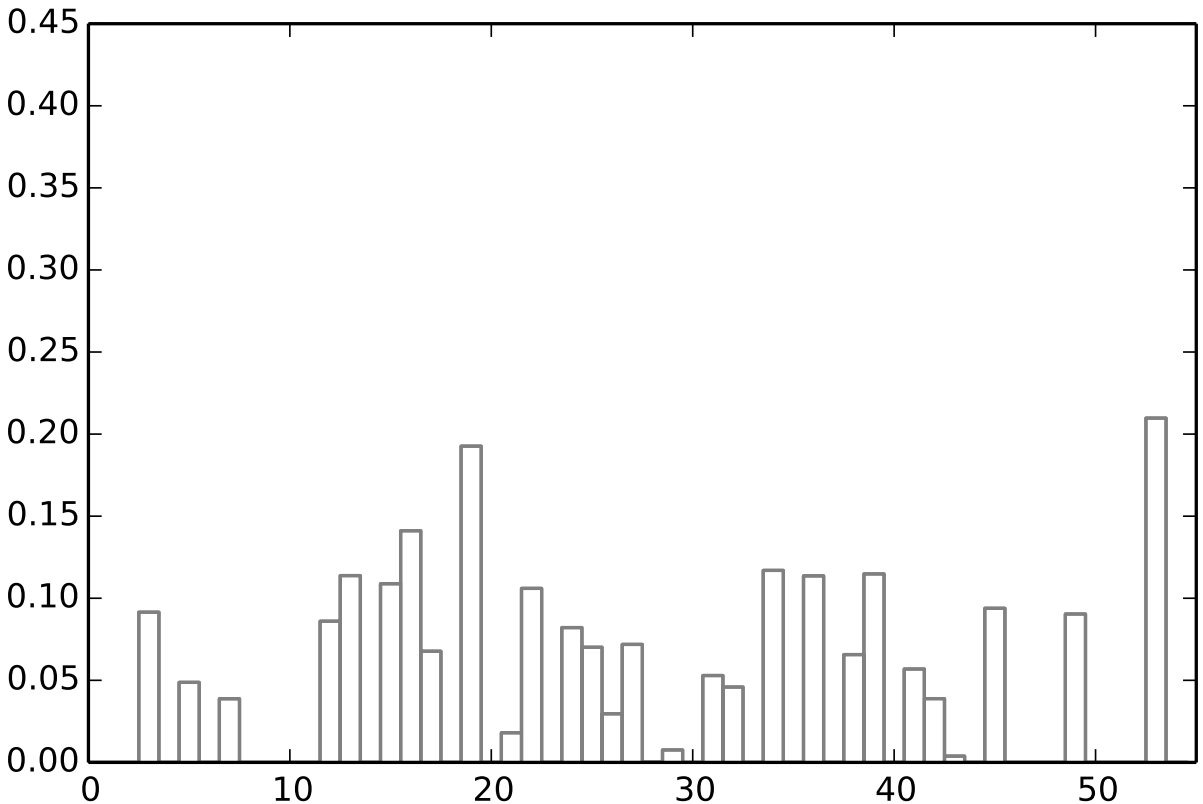
  - Set $\mathcal{I} \leftarrow \mathcal{I}'$
- Output the flattening of q over the intervals in $\mathcal{I}$.

Example ($k = 2$)

Input:



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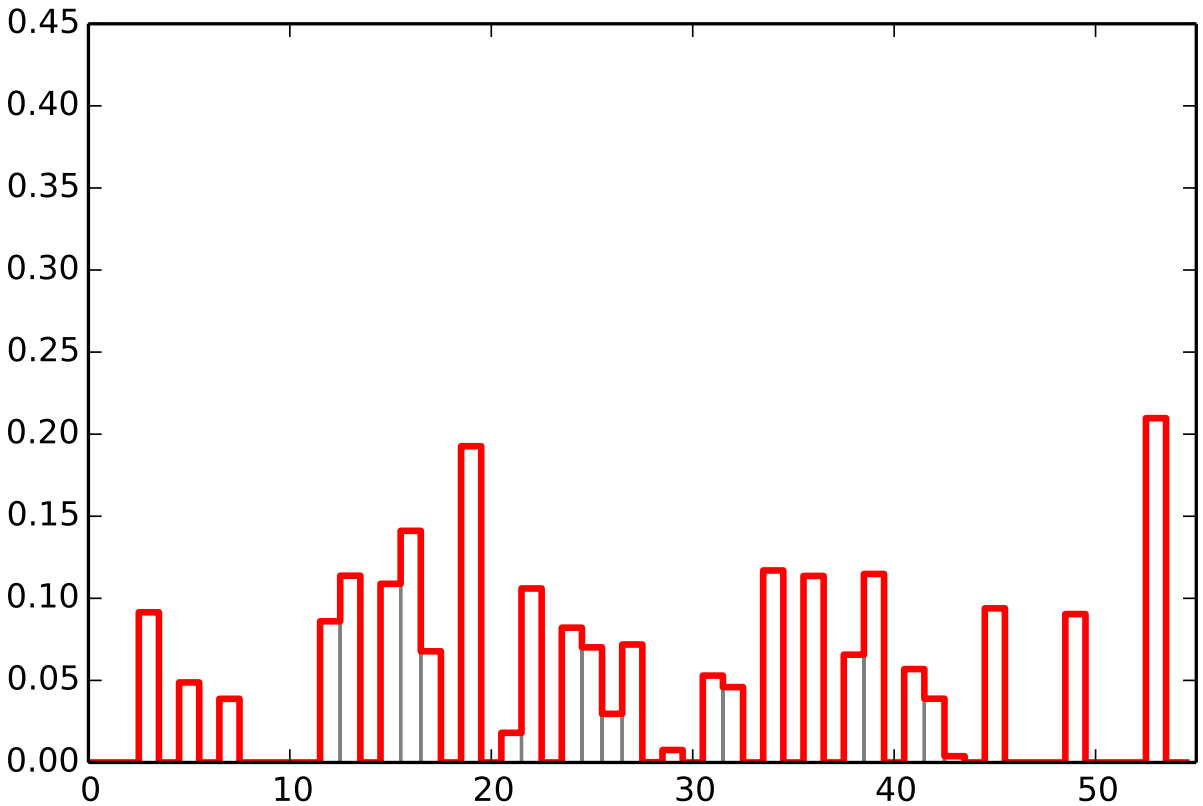
Iteration 0:



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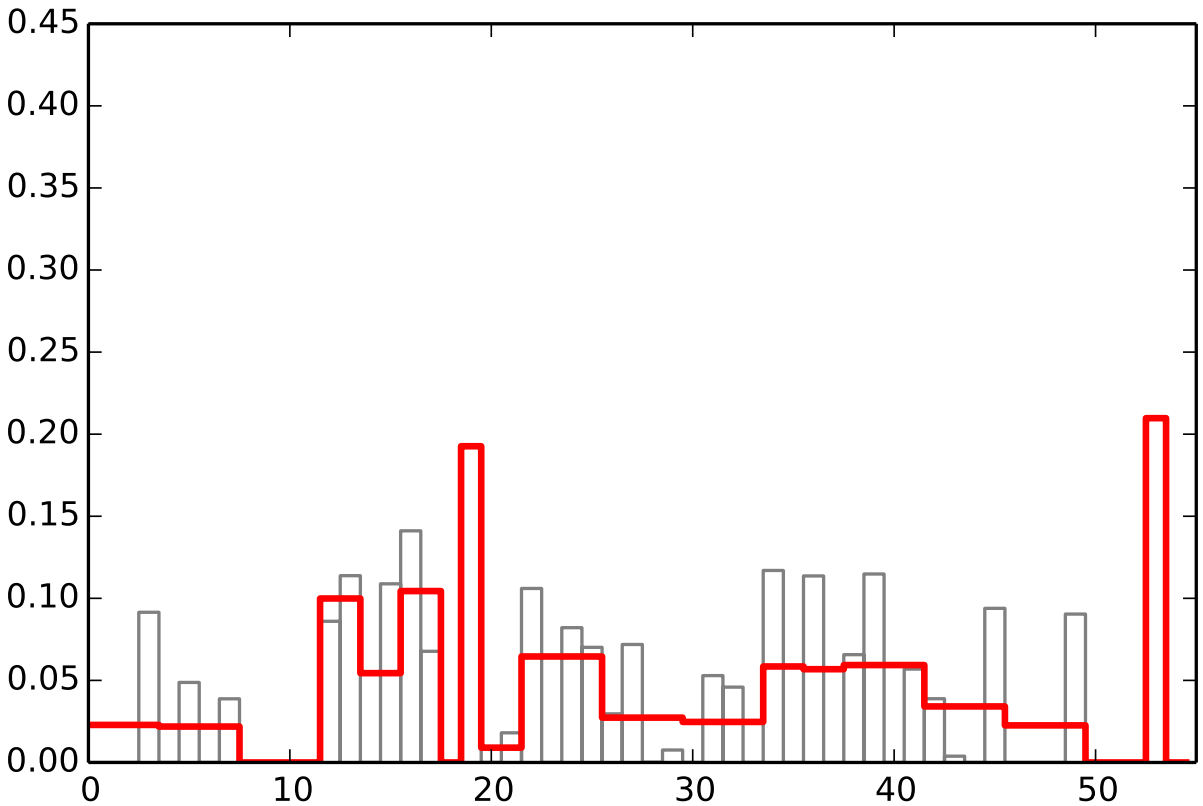
Iteration i:



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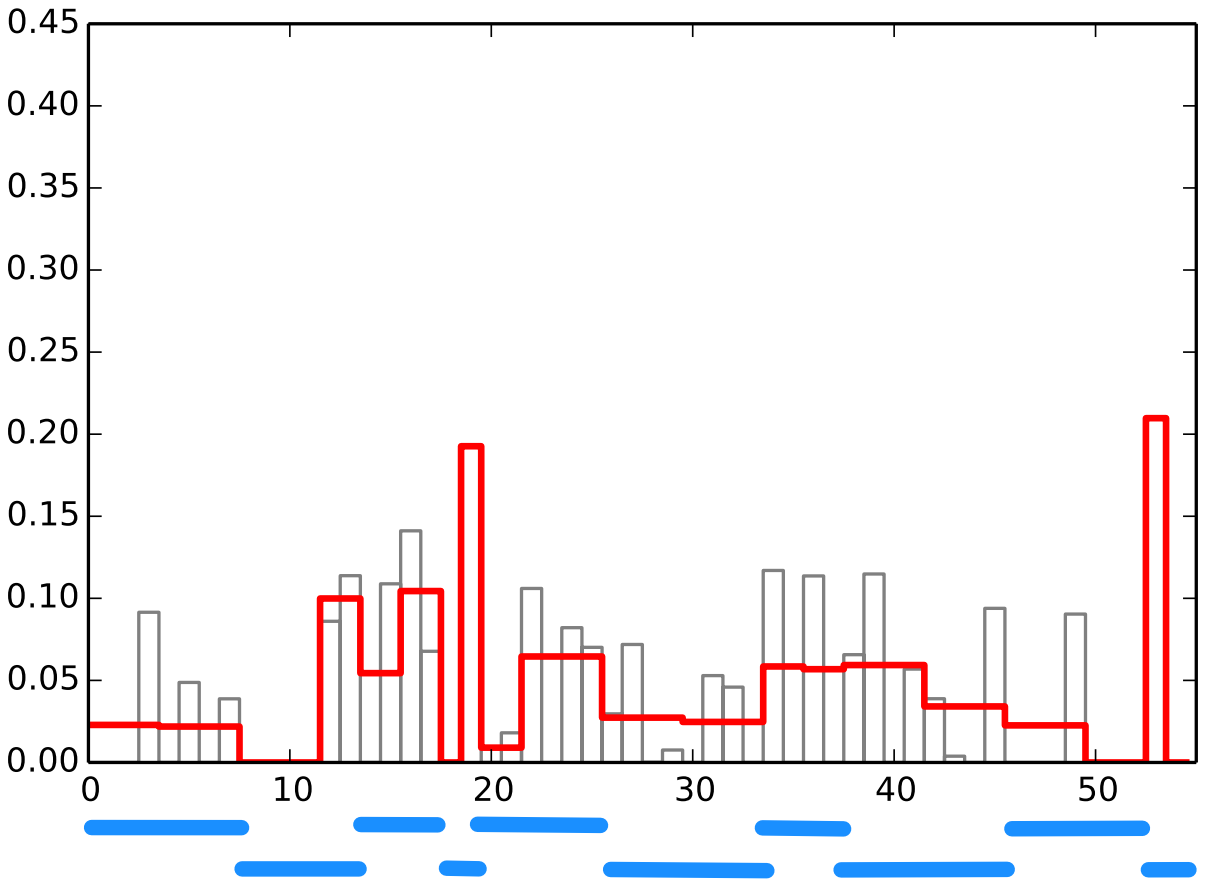

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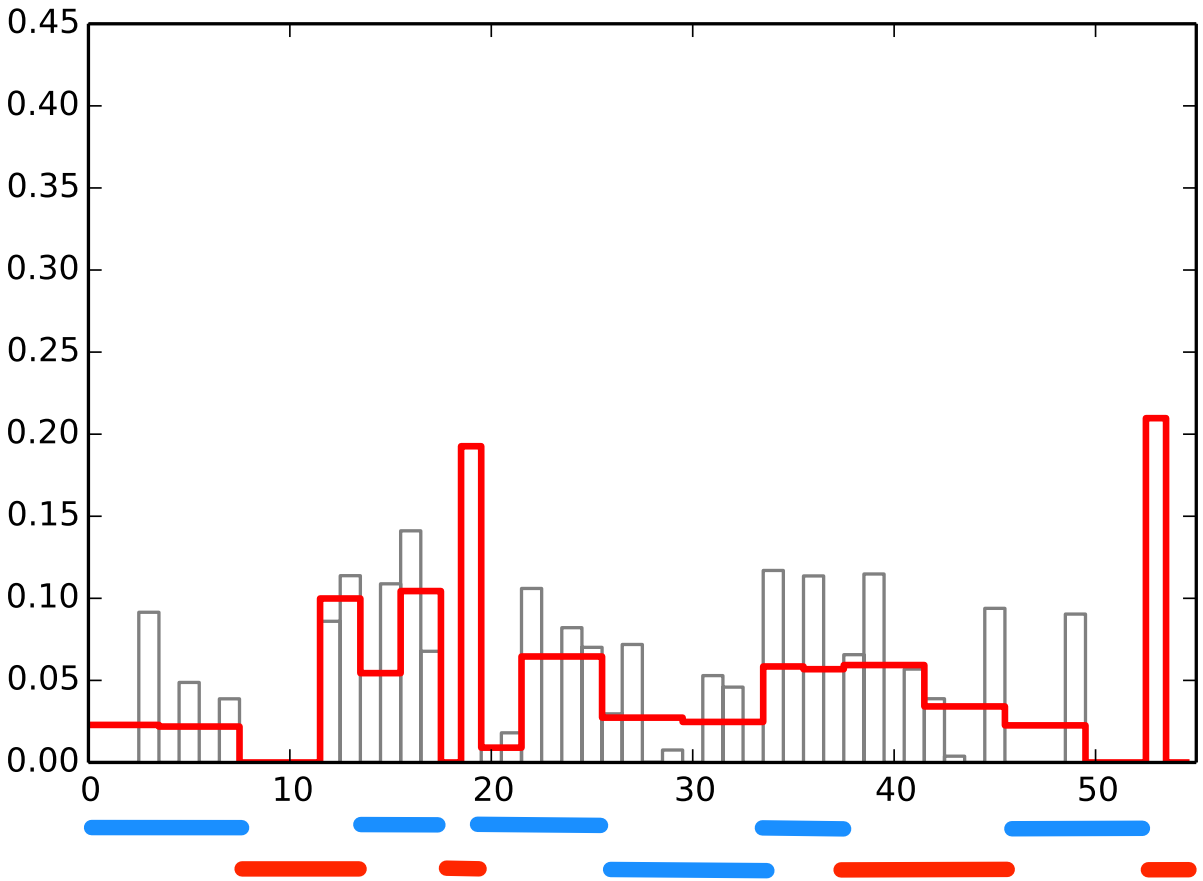
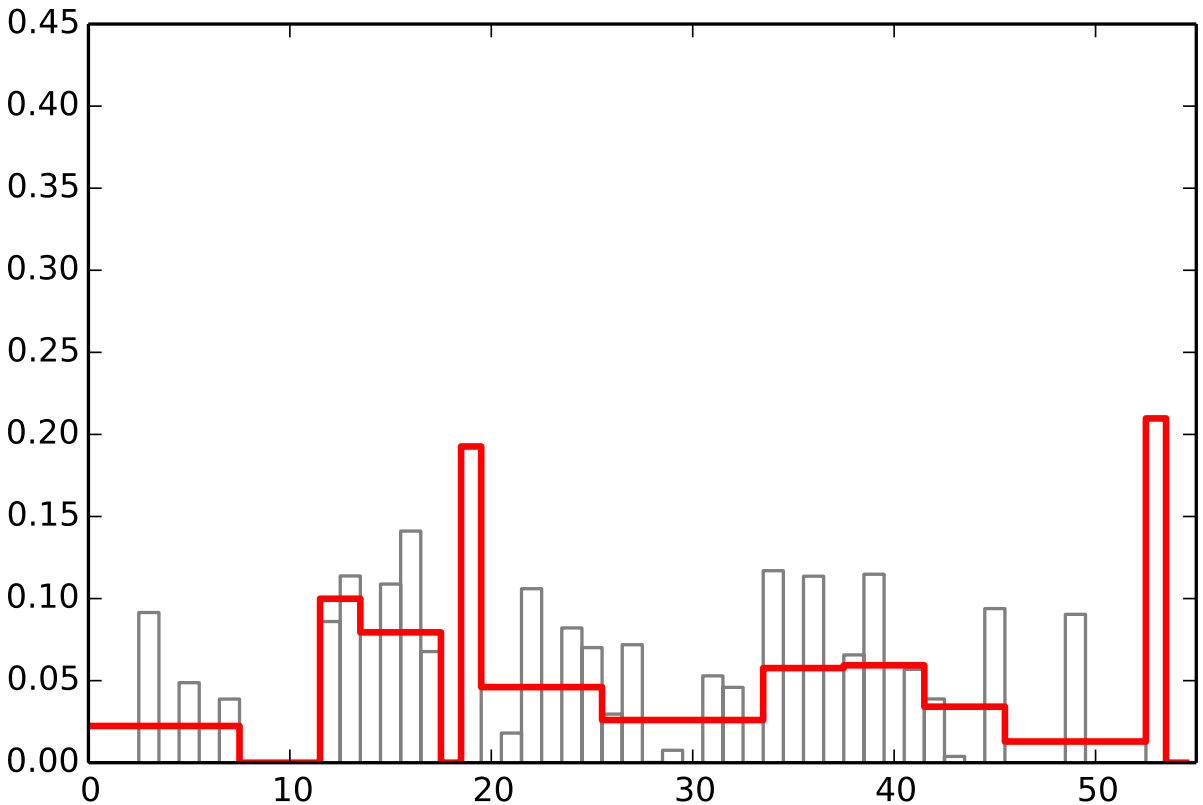


Example ($k = 2$)

Iteration $i + 1$:



Outline of Rest of Talk

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The greedy merging algorithm runs in time $O(m)$.

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Proof Sketch.

- Each iteration can be performed in time proportional to the number of intervals left in the partition in that iteration.
- Let s_j be the number of intervals after the jth iteration of the algorithm. Then

$$s_{j+1} = \frac{s_j - 4k}{2} + 4k = \frac{s_j + 4k}{2} \leq \frac{9}{10} s_j$$

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  as long as $s_j \geq 5k$.
- Thus the runtime is dominated by the runtime of the first iteration, which runs in time $O(m)$. □

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Let h be the output of our algorithm. Then $\|h - q\|_2^2 \leq 2 \cdot OPT_k(q)$.

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We will bound each term separately.

Error Analysis (cont.)

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Notice this is true for any set of intervals on which h^* is flat.

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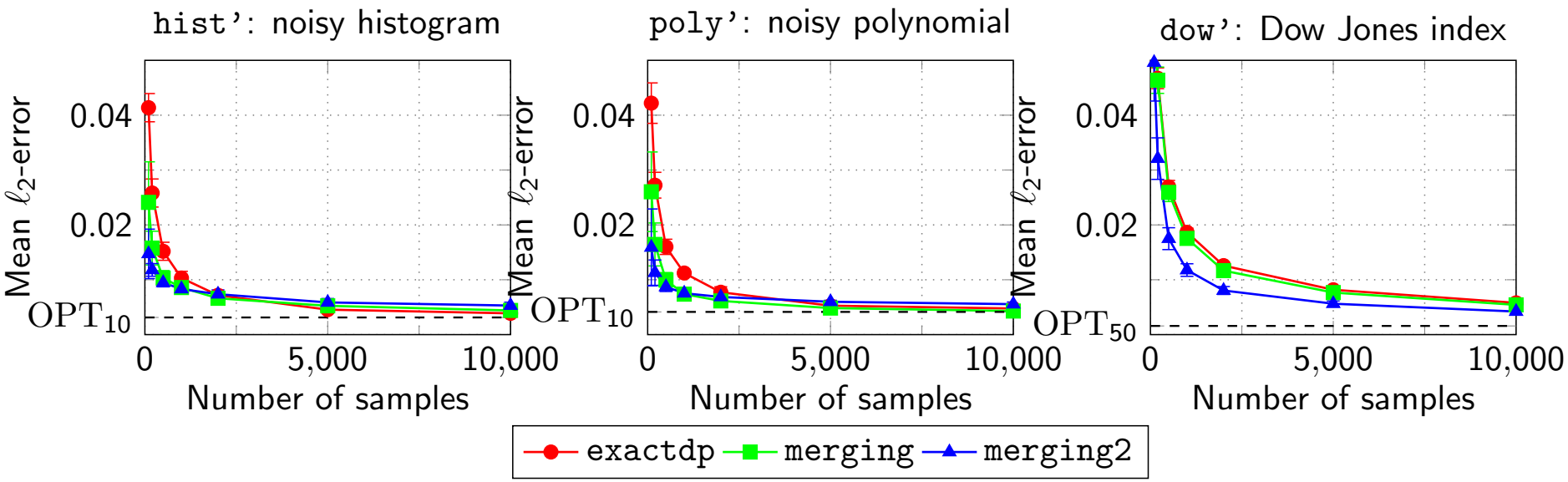
- h^* has at most k jumps $\Rightarrow$ it has no jumps in at least k of the $J_1, \ldots, J_{2k}$. WLOG assume $J_1, \ldots, J_k$ have no jumps of h^*.
- Hence $\sum_{\ell=1}^{k} \text{flat-err}_q(J_\ell) \leq \text{OPT}_k(q) \Rightarrow \text{flat-err}_q(I) \leq \frac{1}{k}\text{OPT}_k(q)$.
- So $\sum_{I \in \mathcal{J}} \text{flat-err}_q(I) \leq |\mathcal{J}| \frac{1}{k}\text{OPT}_k(q) \leq \text{OPT}_k(q)$. □

Outline of Rest of Talk

Error Rate



`exactdp`: Exact DP algorithm for k-histograms.
`merging`: Merging with parameters set to produce a $2k + 1$ histogram.
`merging2`: Merging with parameters set to produce a $k + 1$ histogram.

Experimental Evaluation

		exactdp	merging	merging2	fastmerging	fastmerging2	dual
hist	Error (ℓ_2)	16.1	16.4	16.6	17.0	21.5	25.8
	Error (relative)	**1.00**	**1.02**	**1.03**	**1.06**	**1.34**	**1.60**
	Time (milliseconds)	55.391	0.038	0.037	0.020	0.014	0.108
	Time (relative)	**3,910**	**2.7**	**2.6**	**1.4**	**1.0**	**7.6**
poly	Error (ℓ_2)	105.1	85.9	111.6	85.6	111.7	124.0
	Error (relative)	**1.00**	**0.82**	**1.06**	**0.81**	**1.06**	**1.18**
	Time (milliseconds)	858.064	0.112	0.112	0.048	0.041	0.446
	Time (relative)	**20,924**	**2.7**	**2.7**	**1.2**	**1.0**	**10.9**
dow	Error (ℓ_2)	904.0	733.1	1,046.1	727.5	1,079.1	1,838.1
	Error (relative)	**1.00**	**0.81**	**1.16**	**0.80**	**1.19**	**2.03**
	Time (milliseconds)	73576.921	0.510	0.478	0.205	0.173	1.849
	Time (relative)	**425,540**	**3.0**	**2.8**	**1.2**	**1.0**	**10.7**

Outline of Rest of Talk

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- What if you don't know what k to pick?

We give a hierarchical version of our algorithm which produces good histogram approximations for all values of $k \geq 1$.

- What if you want more powerful representations?

We give a natural generalization of our algorithm which produces good piecewise polynomial approximations.

Extensions of the Algorithm

- What if you don't know what k to pick?

We give a hierarchical version of our algorithm which produces good histogram approximations for all values of $k \geq 1$.

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We give a natural generalization of our algorithm which produces good piecewise polynomial approximations.

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[ADLS15] develops a similar (but more complicated) algorithm for approximation in ℓ_1 (or total variation distance).

Conclusions

- We give the first sample-optimal, linear time algorithm for learning histogram approximations in ℓ_2^2 error.
- Reduce to giving a linear time algorithm for histogram approximation of sparse distributions.
- Do so via a simple, novel greedy merging algorithm.

Open problems:

- Streaming variants? Parallel variants?
- Is our error analysis tight?
- Can we get better α, β?

Thank you!